

# Two Different Definitions on Genuine Quantum and Classical Correlations in Multipartite Systems Do Not Coincide in General

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Giorgi et al [1] presented an approach to study genuine quantum and classical correlations in multipartite systems. They defined total information  $T(\varrho)$ , total classical correlation  $\mathcal{J}(\varrho)$ , and total quantum discord  $\mathcal{D}(\varrho) = T(\varrho) - \mathcal{J}(\varrho)$  for a tripartite state  $\varrho$ , and the genuine tripartite correlation  $T^{(3)}(\varrho) = T(\varrho) - T^{(2)}(\varrho)$ , where  $T^{(2)}(\varrho) = \max[\mathcal{I}(\varrho_{a,b}), \mathcal{I}(\varrho_{a,c}), \mathcal{I}(\varrho_{b,c})]$  with  $\mathcal{I}$  being the bipartite mutual information in  $\varrho$ . Without loss of generality, suppose  $T^{(3)}(\varrho) = T(\varrho) - \mathcal{I}(\varrho_{a,b})$ . Then they defined the concerned genuine tripartite classical and quantum correlations, i.e.,  $\mathcal{J}^{(3)}(\varrho)$  and  $\mathcal{D}^{(3)}(\varrho)$ , in two different ways (labeled by single and double primes respectively): 1)  $T^{(3)}(\varrho) = \mathcal{J}^{(3)'}(\varrho) + \mathcal{D}^{(3)'}(\varrho)$ , where  $T^{(3)}(\varrho)$  is the bipartite mutual information between subsystems  $ab$  and  $c$ ; and 2)

$$\begin{aligned}\mathcal{J}^{(3)''}(\varrho) &= \mathcal{J}(\varrho) - \mathcal{J}^{(2)}(\varrho), \\ \mathcal{D}^{(3)''}(\varrho) &= \mathcal{D}(\varrho) - \mathcal{D}^{(2)}(\varrho),\end{aligned}\quad (1)$$

where  $\mathcal{J}^{(2)}(\varrho) = \max[\mathcal{J}(\varrho_{a,b}), \mathcal{J}(\varrho_{a,c}), \mathcal{J}(\varrho_{b,c})]$  and  $\mathcal{D}^{(2)}(\varrho) = \max[\mathcal{D}(\varrho_{a,b}), \mathcal{D}(\varrho_{a,c}), \mathcal{D}(\varrho_{b,c})]$ . The authors “must show that the two definitions coincide”. However they specialized on the case of three-qubit pure states and shew the coincidence in this case. Nonetheless, we find that the two definitions do not coincide in general.

We first remark that  $\mathcal{D}^{(2)}(\varrho)$  in Ref.[1] is mistakenly written as  $\mathcal{D}^{(2)}(\varrho) = \min[\mathcal{D}(\varrho_{a,b}), \mathcal{D}(\varrho_{a,c}), \mathcal{D}(\varrho_{b,c})]$ . According to the sentence above the equation (4) in Ref.[1], the minimization should be maximization. Otherwise, the two definitions disagree even for three-qubit pure states. Because for a three-qubit pure state  $\varrho$ ,  $\mathcal{D}(\varrho_{a,b}) = \mathcal{I}(\varrho_{a,b}) - \mathcal{J}(\varrho_{a,b}) = S(\varrho_a) - S(\varrho_c) + \mathcal{E}(\varrho_{b,c})$  and  $\mathcal{D}(\varrho_{a,b}) \geq \max[\mathcal{D}(\varrho_{a,c}), \mathcal{D}(\varrho_{b,c})]$  in the case of  $\mathcal{I}(\varrho_{a,b}) \geq \mathcal{I}(\varrho_{a,c}) \geq \mathcal{I}(\varrho_{b,c})$ . Hence,  $\mathcal{D}(\varrho_{a,b})$  is not the minimal bipartite quantum discord. Alternatively, the second equality of equation (8) in Ref.[1] does not hold, i.e.,  $\mathcal{D}^{(2)}(\varrho) \neq S(\varrho_a) - S(\varrho_c) + \mathcal{E}(\varrho_{b,c})$ . As a result, the equation (9) in Ref.[1] does not hold according to the second definition, but does hold according to the first definition.

Let us now consider a three-qubit mixed state  $\varrho = \frac{1}{2}[(|0\rangle\langle 0|)_b \otimes \varrho_{a,c}(0.1, 3\pi/10) + (|1\rangle\langle 1|)_b \otimes \varrho_{a,c}(0.7, \pi/5)]$ , where  $\varrho_{a,c}(p, \theta) = [(1-p)|00\rangle\langle 00| + p\sin^2\theta|01\rangle\langle 01| + p\sin\theta\cos\theta|01\rangle\langle 10| + p\sin\theta\cos\theta|10\rangle\langle 01| + p\cos^2\theta|10\rangle\langle 10|]_{a,c}$ . One has  $\mathcal{I}(\varrho_{a,b}) \approx 0.27$ ,  $\mathcal{I}(\varrho_{a,c}) \approx 0.22$ ,  $\mathcal{I}(\varrho_{b,c}) \approx 0.01$ . Hence,  $T^{(2)}(\varrho) = \mathcal{I}(\varrho_{a,b})$ . Note that  $\varrho$  is actually a bipartite classical-quantum correlated state in the bipartite partition  $b$  and  $ac$ . Consequently, in terms of Refs. [2, 3] one

can get  $\mathcal{D}(\varrho_{a,b}) = \mathcal{D}(\varrho_{b,c}) = 0$ , i.e.,  $\mathcal{J}(\varrho_{a,b}) = \mathcal{I}(\varrho_{a,b})$  and  $\mathcal{J}(\varrho_{b,c}) = \mathcal{I}(\varrho_{b,c})$ . Moreover,  $\varrho_{a,c}$  is actually a bipartite entangled state, since the entanglement of formation [4] of  $\varrho_{a,c}$  is positive,  $\mathcal{E}(\varrho_{a,c}) \approx 0.11 > 0$ . Consequently,  $\mathcal{D}(\varrho_{a,c}) > 0$  and  $\mathcal{J}(\varrho_{a,c}) < \mathcal{I}(\varrho_{a,c})$ . Therefore  $\mathcal{J}^{(2)}(\varrho)$  is just  $\mathcal{J}(\varrho_{a,b})$  and  $\mathcal{D}^{(2)}(\varrho)$  is  $\mathcal{D}(\varrho_{a,c})$ . At last we have  $\mathcal{J}^{(3)''}(\varrho) + \mathcal{D}^{(3)''}(\varrho) = [\mathcal{J}(\varrho) - \mathcal{J}^{(2)}(\varrho)] + [\mathcal{D}(\varrho) - \mathcal{D}^{(2)}(\varrho)] = T(\varrho) - \mathcal{J}^{(2)}(\varrho) - \mathcal{D}(\varrho_{a,c}) = T^{(3)}(\varrho) - \mathcal{D}(\varrho_{a,c})$ . In contrast, we get  $\mathcal{J}^{(3)'}(\varrho) + \mathcal{D}^{(3)'}(\varrho) = T^{(3)}(\varrho)$ . Obviously, both definitions on  $\mathcal{J}^{(3)}(\varrho)$  and  $\mathcal{D}^{(3)}(\varrho)$  disagree for this simple example.

Generalizing to the multipartite case, the authors further stated that for  $n$ -partite pure states it is still possible to define, through a ladder procedure,  $\mathcal{D}^{(n)}$  and  $\mathcal{J}^{(n)}$  in analogy with Eq.(1). Since the coincidence can not be completely assured for all tripartite states, the coincidence of the two definitions after the generalization via the so-called ladder procedure to multipartite cases is again not reliable. Incidentally, for an  $n$  ( $n \geq 4$ )-partite pure state, its reduced tripartite states are mixed ones in general. For instance, as the reduced state of the 6-qubit pure state  $|\Psi\rangle_{abca'b'c'} = \{|00\rangle_{bb'}[\sqrt{0.9}|00\rangle_{ac}|00\rangle_{a'c'} + \sqrt{0.1}(\sin\frac{3\pi}{10}|01\rangle_{ac} + \cos\frac{3\pi}{10}|10\rangle_{ac})|01\rangle_{a'c'}] + |11\rangle_{bb'}[\sqrt{0.7}|00\rangle_{ac}|10\rangle_{a'c'} + \sqrt{0.3}(\sin\frac{\pi}{5}|01\rangle_{ac} + \cos\frac{\pi}{5}|10\rangle_{ac})|11\rangle_{a'c'}]\}/\sqrt{2}$ ,  $\rho_{abc} = \text{Tr}_{a'b'c'}(|\Psi\rangle\langle\Psi|)_{abca'b'c'}$  is just the tripartite mixed state in our example above in revealing the disagreement.

In summary, the genuine quantum and classical correlations defined by Giorgi et al [1] in two different ways do not coincide in general.

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